

Year 8 Maths · Unit 8.1 Fractions, Decimals and Percentages

The big idea. Equivalence and relationships between fractions, decimals and percentages: write quantities in different forms, convert between them, and calculate percentage increase and decrease.

Key vocabulary

Word	Meaning
Numerator	the number above the line in a fraction: how many parts you have
Denominator	the number below the line in a fraction: how many equal parts the whole is cut into
Equivalent fractions	fractions that name the same amount, e.g. $\frac{1}{2}$ and $\frac{3}{6}$
Highest common factor (HCF)	the biggest number that divides exactly into two or more numbers
Lowest terms	a fraction written so that the numerator and denominator share no common factor except 1
Improper fraction	a fraction whose numerator is at least as big as its denominator, e.g. $\frac{7}{4}$
Mixed numeral	a whole number and a fraction together, e.g. $1\frac{3}{4}$
Common denominator	a denominator shared by two or more fractions, so they can be compared, added or subtracted
Reciprocal	a fraction turned upside down: the reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$
Per cent	per hundred: 75% means $\frac{75}{100}$
Terminating decimal	a decimal that ends, e.g. $\frac{1}{8} = 0.125$
Recurring decimal	a decimal with digits that repeat forever, e.g. $\frac{1}{3} = 0.333...$
Multiplier	the number you multiply by to increase or decrease an amount by a percentage, e.g. $\times 1.2$ for a 20% increase

Forms of fractions

- **Equivalent:** multiply or divide numerator and denominator by the same number: $\frac{1}{2} = \frac{3}{6}$.
- **Simplify:** divide the numerator and denominator by the HCF: $\frac{12}{18} = \frac{2}{3}$.
- **Improper to mixed:** $17 \div 5 = 3$ remainder 2, so $\frac{17}{5} = 3\frac{2}{5}$. **Mixed to improper:** $2\frac{1}{3} = (\frac{3 \times 2}{3} + \frac{1}{3}) = \frac{7}{3}$.
- **Compare:** common denominator first. $\frac{3}{4} = \frac{9}{12} > \frac{2}{3} = \frac{8}{12}$.
- Why it works: you cut the pieces the same way everywhere, so the amount does not change.

Four operations with fractions

- **Add/subtract:** common denominator, then add or subtract the numerators. $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$.
- **Never** add the denominators: $\frac{1}{2} + \frac{1}{3}$ is not $\frac{2}{5}$.
- **Multiply:** numerator $\times$ numerator, denominator $\times$ denominator (cancel first). $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$.
- **Divide:** multiply by the reciprocal. $3 \div \frac{1}{4} = 3 \times 4 = 12$ (how many quarters fit in 3?).
- Mixed numerals: change to improper first. $2 \frac{1}{2} \div \frac{5}{6} = \frac{5}{2} \times \frac{6}{5} = 3$.

Fractions, decimals and percentages

- Fraction to decimal: divide numerator by denominator ($\frac{3}{4} = 0.75$). Decimal to percentage: $\times 100$. Percentage to decimal: $\div 100$. Percentage to fraction: over 100, then simplify.
- Per cent means per hundred: $\frac{3}{4} = \frac{75}{100} = 0.75 = 75\%$.
- Common ones: $\frac{1}{2} = 0.5 = 50\%$; $\frac{1}{4} = 0.25 = 25\%$; $\frac{1}{5} = 0.2 = 20\%$; $\frac{1}{10} = 0.1 = 10\%$; $\frac{1}{8} = 0.125 = 12.5\%$.
- **Terminating** decimals end ($\frac{1}{8} = 0.125$); **recurring** decimals repeat ($\frac{1}{3} = 0.333\dots$). A fraction in lowest terms terminates only if its denominator has no prime factors except 2 and 5.
- To order mixed forms, change them all to percentages.

Percentage increase and decrease

- **Multiplier** = $1 +$ the rise, or $1 -$ the fall, as a decimal. $+20\%$ is $\times 1.2$; -15% is $\times 0.85$.
- Why 1.2 and not 0.2? The original is 100%, and $100\% + 20\% = 120\%$.
- 45 francs up 20%: $45 \times 1.2 = 54$. 300 francs down 15%: $300 \times 0.85 = 255$.
- **Repeated change:** multiply the multipliers. Up 20% then down 20% = $1.2 \times 0.8 = 0.96$, which is 4% less than the start (the second 20% is of a bigger amount).
- Check by estimating: a 20% rise on 45 is about 9 more.

Common mistakes

- $\times$ “ $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$.” $\checkmark$ Find a common denominator first: $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$.
- $\times$ “ $3 \div \frac{1}{4} = \frac{3}{4}$.” $\checkmark$ Dividing by $\frac{1}{4}$ multiplies by 4: $3 \times 4 = 12$.
- $\times$ “A 20% rise is $\times 0.2$.” $\checkmark$ It is $\times 1.2$, because the original stays ($100\% + 20\%$).
- $\times$ “Up 20% then down 20% gets you back to the start.” $\checkmark$ It gives 96% of the start: $1.2 \times 0.8 = 0.96$.
- $\times$ “Leave $\frac{12}{18}$ as it is.” $\checkmark$ Simplify to lowest terms: $\frac{2}{3}$.