

UNIT 8.2

Ratio and Proportion

Revision slides: everything you need for the test, one idea per slide.

Scale copies and similarity

Two shapes are **similar** if one is an enlargement of the other: matching angles are equal and matching lengths are in the same ratio.

Scale factor = new length $\div$ old length. A 3 cm side becoming 12 cm has scale factor $12 \div 3 = 4$.

Use the scale factor to find a missing length: multiply by it going bigger, divide going smaller.

Stretching in one direction only does **not** give a copy: the shape is distorted.

Ratio (1 of 2)

A **ratio** compares two quantities: $3 : 2$. **Proportion** compares a part with the whole: $3/5$.

Total parts = add the ratio numbers. To share 40 in the ratio $3 : 2 : 5$ parts, 1 part = 8, so 24 and 16.

Simplify a ratio by dividing both by the HCF: $18 : 24 = 3 : 4$. Use the same units first: $2 \text{ m} : 50 \text{ cm} = 200 : 50 = 4 : 1$.

Ratio (2 of 2)

Equivalent ratios come from multiplying or dividing both parts by the same number.

Form **1 : n** (divide by the first): $4 : 10 = 1 : 2.5$. Form **n : 1** divides by the second. The unit ratio makes comparing easy.

Ratio and fractions

Turn a ratio into fractions of the whole: $3 : 2$ means $\frac{3}{5}$ and $\frac{2}{5}$.

Turn a fraction into a ratio: $\frac{1}{4}$ are girls means girls : others = $1 : 3$.

Part of a quantity: $\frac{3}{5}$ of 40 = 24.

A ratio compares part with part ($3 : 2$); a fraction compares part with whole ($\frac{3}{5}$).

Direct proportion (1 of 2)

Two quantities are in **direct proportion** if when one doubles so does the other. Their graph is a straight line through the origin.

Use the **unit value**: 5 pens cost 15 francs, so 1 pen costs 3 francs, and 8 pens cost 24 francs.

Direct proportion (2 of 2)

The **constant of proportionality** k is $y \div x$. The rule is **$y = kx$** . If $y = 15$ when $x = 5$, $k = 3$ and $y = 3x$.

Test a table: if $y \div x$ is the same every time, it is proportional.

A relationship that does not pass through the origin (for example a taxi with a fixed charge) is **not** proportional.

Conversion graphs

A conversion graph is a straight line through the origin: for example francs to euros.

Read across from one axis to the line, then down (or across) to the other. You can convert both ways.

Draw one from two points (the origin and one known pair) and join them.

If the line does not go through the origin, the relationship is not proportional.

Scale diagrams and map scales

A scale **1 : 50** means 1 cm on the drawing is 50 cm in real life.

Real length = drawing length $\times$ scale. Drawing length = real length $\div$ scale.

Map scale 1 : 25 000: 4 cm on the map is $4 \times 25\,000 = 100\,000$ cm = 1 km.

Change units sensibly: 100 cm = 1 m and 100 000 cm = 1 km.

Choose a scale that fits the paper and keeps the drawing clear.

WATCH OUT · MISCONCEPTIONS

Don't fall for these

✗ "Share 40 in the ratio 3 : 2 as $40 \div 3$ and $40 \div 2$."

✓ Add the parts (5), find one part (8), then multiply: 24 and 16.

✗ "Ratio 2 m : 50 cm is 2 : 50."

✓ Change to the same units first: $200 : 50 = 4 : 1$.

✗ "Ratio and fraction are the same thing."

✓ 3 : 2 compares part with part; $\frac{3}{5}$ compares part with whole.

✗ "Any straight line graph is proportional."

✓ It must also pass through the origin.

✗ "1 : 25 000 means 1 cm is 25 000 m."

✓ 1 cm is 25 000 cm = 250 m.

TEST YOURSELF

Test yourself

1. A side of 3 cm becomes 12 cm in a similar shape. What is the scale factor?

Answer 4. $12 \div 3 = 4$.

2. A similar rectangle has scale factor 3. A side was 5 cm. What is it now?

Answer 15 cm. $5 \times 3 = 15$ cm.

3. 5 pens cost 15 francs. How much do 8 pens cost?

Answer 24 francs. 1 pen = 3 francs, so 8 pens = 24 francs.

4. Which table is directly proportional?

Answer x: 1, 2, 3 / y: 4, 8, 12. $y \div x = 4$ every time.