

How to decide which method to use

Follow the steps in order. Do not start factorising until you have made one side equal to 0

Make one side equal to 0

1

Write it as $ax^2 + bx + c = 0$. If a is negative, multiply every term by -1

Then look for a **common factor** in every term and take it out first (for example $2x^2 - 8 = 2(x^2 - 4)$)



2

Look at which terms are there

No x term ($b = 0$)

such as $x^2 - 49 = 0$ or $x^2 = 20$

Minus between two squares? Difference of two squares

$$a^2 - b^2 = (a + b)(a - b)$$

Otherwise get x^2 on its own and square root both sides, with $\pm$

If x^2 equals a negative number: no real solutions

No number term ($c = 0$)

such as $x^2 + 7x = 0$

Common factor

$$x(x + 7) = 0$$

$$x = 0 \text{ or } x + 7 = 0$$

Never divide by x : you lose $x = 0$

All three terms are there

such as $x^2 + 6x + 3 = 0$ or $2x^2 + 7x + 3 = 0$

Go to step 3: the discriminant $b^2 - 4ac$

It tells you how many solutions there are, and whether the quadratic will factorise



3

Work out the discriminant $b^2 - 4ac$

Take care with signs: if b is negative, b^2 is still positive. Substitute with brackets

Negative

for example -16

No real solutions

The graph never touches the x -axis.

Stop

Zero

One repeated solution

Perfect square

$$(px + q)^2 = 0$$

such as $x^2 + 6x + 9 = (x + 3)^2$

Positive and a perfect square

such as 25, 49, 81

It factorises

$a = 1$: find two numbers that multiply to c and add to b

$a \neq 1$: the ac method

Positive, not a perfect square

such as 13, 20, 24

It will not factorise nicely

Complete the square (or use the formula)

Answers are surds, such as $-3 \pm \sqrt{6}$

Why the discriminant is useful

A perfect square number is 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169 ...

If the discriminant is one of these, the quadratic has rational roots and factorises. If it is not, do not waste time hunting for factors. Go straight to completing the square

Finish every question by checking

Substitute each solution back into the original equation. Both sides should be equal. Or expand your brackets and see if you get back to the start



Quick patterns and worked decisions

Difference of two squares

$$a^2 - b^2 = (a + b)(a - b)$$

Two terms, a minus sign, both squares

$$4x^2 - 25 = (2x + 5)(2x - 5)$$

$x^2 + 25$ is **not** one

Perfect squares

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

First and last are squares, middle is 2 × the roots

$$x^2 - 10x + 25 = (x - 5)^2$$

Signs when c is positive or negative

c positive: both numbers have the sign of b

c negative: one + and one −, the larger has the sign of b

$$x^2 + 2x - 15 = (x + 5)(x - 3)$$

Worked decisions: what do I notice, and which method?

Equation	What I notice	Method	Solutions
$x^2 + 7x = 0$	No number term	Common factor	$x = 0$ or $x = -7$
$x^2 - 49 = 0$	No x term, minus, two squares	Difference of two squares	$x = 7$ or $x = -7$
$x^2 = 20$	No x term, 20 not a square	Square roots	$x = \pm 2\sqrt{5}$
$x^2 + 10x + 25 = 0$	Discriminant 0	Perfect square	$x = -5$ (one solution)
$x^2 - x - 12 = 0$	Discriminant $49 = 7^2$	Two numbers (−4 and 3)	$x = 4$ or $x = -3$
$2x^2 + 7x + 3 = 0$	Discriminant $25 = 5^2$, and $a \neq 1$	ac method (1 and 6)	$x = -\frac{1}{2}$ or $x = -3$
$x^2 + 6x + 3 = 0$	Discriminant 24, not a square	Complete the square	$x = -3 \pm \sqrt{6}$
$x^2 + 2x + 5 = 0$	Discriminant −16	None needed	No real solutions

The ac method (a is not 1)

$$\text{Solve } 2x^2 + 7x + 3 = 0$$

1 Multiply $a \times c = 6$

2 Two numbers that multiply to 6 and add to 7: 1 and 6

3 Split the middle: $2x^2 + x + 6x + 3$

4 Group: $x(2x + 1) + 3(2x + 1)$

$$\mathbf{5 (2x + 1)(x + 3) = 0}$$

Completing the square

$$\text{Solve } x^2 + 6x + 3 = 0$$

1 Half of 6 is 3

$$2 (x + 3)^2 - 9 + 3 = 0$$

$$3 (x + 3)^2 = 6$$

$$4 x + 3 = \pm \sqrt{6}$$

$$\mathbf{5 x = -3 + \sqrt{6} \text{ or } x = -3 - \sqrt{6}}$$

If a is not 1, take a out of the x terms only

Key vocabulary

Discriminant: $b^2 - 4ac$, the number that tells you how many solutions there are

Perfect square number: 1, 4, 9, 16, 25 ...

Root, solution, zero: a value of x that makes the quadratic equal 0

Factorise: write as a product of brackets

Surd: a root that is not exact, such as $\sqrt{6}$

Common mistakes

Factorising before making one side 0

Dividing by x instead of taking it out

Forgetting $\pm$ when square rooting

Writing b^2 as negative when b is negative

Using a minus sign for $x^2 + 25$: it is not a difference of squares

