

UNIT 11.1.3

Quadratic Functions

Revision slides: everything you need for the test, one idea per slide.

Key vocabulary (1 of 3)

Quadratic function

a function of the form $f(x) = ax^2 + bx + c$ with $a \neq 0$

Parabola

the curved graph of a quadratic function

Vertex

the turning point of a parabola: a minimum or a maximum

Axis of symmetry

the vertical line through the vertex; the graph is the same on both sides

Key vocabulary (2 of 3)

y-intercept

the point where the graph crosses the y-axis ($x = 0$)

x-intercept (zero)

a point where the graph crosses the x-axis ($y = 0$)

Standard form

$$y = ax^2 + bx + c$$

Factorised form

$$y = a(x - p)(x - q)$$

Key vocabulary (3 of 3)

Vertex form

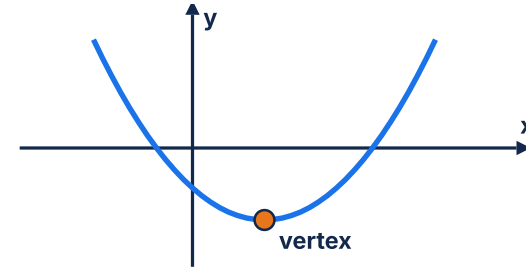
$$y = a(x - h)^2 + k$$

Domain and range

the allowed inputs (x) and the outputs (y) of the function

Quadratic functions and parabolas

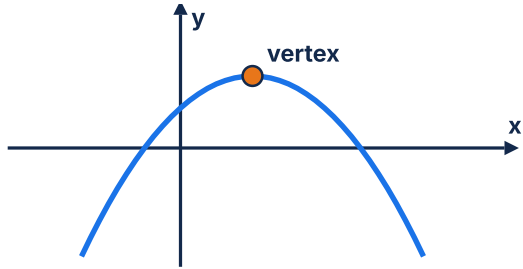
A **quadratic function** has the form $f(x) = ax^2 + bx + c$, where $a \neq 0$.
Its graph is a curve called a **parabola**.



If $a > 0$ the parabola opens **upwards** (a U shape) and the vertex is a **minimum**.

| **EXAMPLE** $f(x) = x^2 - 4x + 5$

Quadratic functions and parabolas (continued)



If $a < 0$ the parabola opens **downwards** (a $\cap$ shape) and the vertex is a **maximum**.

EXAMPLE $f(x) = -x^2 + 2x + 3$

CHECK 1 Does $f(x) = 3 - 2x^2$ have a maximum or a minimum?

The size of a changes the width.

A bigger value of a (ignoring the sign) makes the parabola narrower.

EXAMPLE $y = 3x^2$ is narrower than $y = x^2$

EXAMPLE $y = \frac{1}{2}x^2$ is wider than $y = x^2$

Intercepts and the axis of symmetry

The **y-intercept** is where $x = 0$.
In standard form it is the constant term c .

EXAMPLE $f(x) = x^2 - 6x + 5$ has y-intercept $(0, 5)$

The **x-intercepts** (zeros, roots) are where $y = 0$.

Solve the quadratic equation by factorising, using the formula or using your GDC.

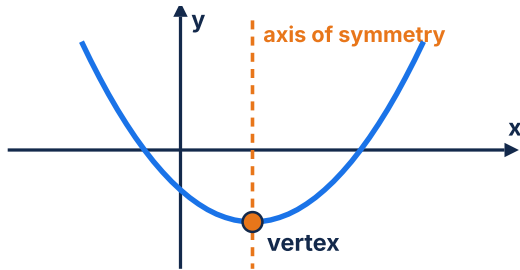
EXAMPLE $x^2 - 6x + 5 = 0$

EXAMPLE $(x - 1)(x - 5) = 0$

EXAMPLE $x = 1$ or $x = 5$

EXAMPLE There can be 0, 1 or 2 x-intercepts

Intercepts and the axis of symmetry (continued)



The **axis of symmetry** is a vertical line through the vertex.

Its equation is $x = -\frac{b}{2a}$.

It is halfway between the x-intercepts.

| **EXAMPLE** $f(x) = x^2 - 6x + 5$

| **EXAMPLE** $x = -\frac{-6}{2(1)} = 3$

The **vertex** lies on the axis of symmetry.

Substitute the axis value into $f(x)$ to find the y-coordinate.

| **EXAMPLE** $f(3) = 9 - 18 + 5 = -4$

| **EXAMPLE** vertex $(3, -4)$

The three forms of a quadratic

Standard form: $y = ax^2 + bx + c$.

It shows the **y-intercept** c .

| **EXAMPLE** $y = x^2 - 6x + 5$

Factorised form: $y = a(x - p)(x - q)$.

It shows the **x-intercepts** p and q .

| **EXAMPLE** $y = (x - 1)(x - 5)$

| **EXAMPLE** x-intercepts at 1 and 5

| **EXAMPLE** Beware the signs: $(x + 2)$
gives an x-intercept of -2

The three forms of a quadratic (continued)

Vertex form: $y = a(x - h)^2 + k$.

It shows the **vertex** (h, k) and the axis of symmetry $x = h$.

| **EXAMPLE** $y = (x - 3)^2 - 4$

| **EXAMPLE** vertex $(3, -4)$

| **EXAMPLE** Beware the sign: $(x + 2)^2$
gives $h = -2$

All three forms describe the **same** graph.

The value of a is the same in each form.

| **EXAMPLE** $y = x^2 - 6x + 5 = (x - 1)(x - 5) = (x - 3)^2 - 4$

Converting between the forms

Factorised to standard: expand the brackets.

| **EXAMPLE** $y = (x - 1)(x - 5)$

| **EXAMPLE** $= x^2 - 5x - x + 5$

| **EXAMPLE** $= x^2 - 6x + 5$

Standard to factorised: solve $ax^2 + bx + c = 0$ to find p and q, then write $y = a(x - p)(x - q)$.

| **EXAMPLE** $y = x^2 - 6x + 5$

| **EXAMPLE** zeros 1 and 5

| **EXAMPLE** $y = (x - 1)(x - 5)$

Standard to vertex: find $h = -\frac{b}{2a}$, then $k = f(h)$, then write $y = a(x - h)^2 + k$.

| **EXAMPLE** $y = 2x^2 - 12x + 13$

| **EXAMPLE** $h = -\frac{-12}{4} = 3$

| **EXAMPLE** $k = 2(9) - 36 + 13 = -5$

| **EXAMPLE** $y = 2(x - 3)^2 - 5$

Converting between the forms (continued)

Vertex to standard: expand the bracket and collect terms.

| **EXAMPLE** $y = 2(x - 3)^2 - 5$

| **EXAMPLE** $= 2(x^2 - 6x + 9) - 5$

| **EXAMPLE** $= 2x^2 - 12x + 13$

Factorised to vertex: the axis is $x = \frac{p + q}{2}$, so $h = \frac{p + q}{2}$. Substitute h to find k .

| **EXAMPLE** $y = (x - 1)(x - 5)$

| **EXAMPLE** $h = 3$ and $k = (2)(-2) = -4$

| **EXAMPLE** $y = (x - 3)^2 - 4$

CHECK 3 Write $y = (x - 2)(x + 4)$ in standard form

Which form to use

Use **standard form** to read the y-intercept and to type into a calculator.

EXAMPLE the y-intercept of $y = x^2 - 6x + 5$ is 5

Use **factorised form** to read the x-intercepts (zeros) straight away.

EXAMPLE $y = (x - 1)(x - 5)$ crosses the x-axis at 1 and 5

Which form to use (continued)

Use **vertex form** to read the maximum or minimum and where it happens.

EXAMPLE $y = (x - 3)^2 - 4$ has a minimum value of -4 at $x = 3$

Vertex form also shows the **transformations** of $y = x^2$.

$y = a(x - h)^2 + k$ is a stretch of factor a , then a translation of h units right and k units up.

EXAMPLE $y = (x - 3)^2 - 4$ is $y = x^2$ moved 3 right and 4 down

Domain and range

The **domain** of a quadratic function is all real numbers, $x \in \mathbb{R}$, unless the question restricts it.

The **range** depends on the vertex.

If $a > 0$: $y \geq k$.

If $a < 0$: $y \leq k$.

Here k is the y -coordinate of the vertex.

EXAMPLE $y = (x - 3)^2 - 4$ has range $\{y \in \mathbb{R} : y \geq -4\}$

EXAMPLE $y = 5 - x^2$ has range $\{y \in \mathbb{R} : y \leq 5\}$

Domain and range (continued)

With a **restricted domain**, check the vertex and both end points.

| **EXAMPLE** $f(x) = x^2 - 6x + 5$ for $0 \leq x \leq 4$

| **EXAMPLE** $f(0) = 5$, $f(4) = -3$, vertex $(3, -4)$

| **EXAMPLE** range $\{y \in \mathbb{R} : -4 \leq y \leq 5\}$

CHECK 4 State the range of $f(x) = x^2 + 2$

Sketching a quadratic by hand

Step 1: decide the shape from the sign of a .

EXAMPLE Step 2: find the y -intercept ($x = 0$).

EXAMPLE Step 3: find the x -intercepts ($y = 0$), if there are any.

Step 4: find the axis of symmetry and the vertex.

Step 5: draw a smooth curve through the points, symmetrical about the axis.

Label the intercepts and the vertex with coordinates.

EXAMPLE The point $(0, 5)$ on $y = x^2 - 6x + 5$ has a mirror point $(6, 5)$

GDC skills: TI-Nspire

Graph it: open a **Graphs** page and type the function into $f1(x)$.

Press **enter**.

Adjust the window with **menu** → **Window / Zoom** until you can see the intercepts and the vertex.

Features of the graph: menu → **Analyze Graph**.

Choose **Zero** (x-intercepts), **Minimum** or **Maximum** (vertex).

Move the pointer to the left of the point, press enter, then to the right and press enter.

Table of values: menu → **Table** → **Split-screen Table** (or ctrl + T).

Use it to check values and to see the symmetry.

GDC skills: TI-Nspire (continued)

Intersections: type a second function into $f2(x)$, then **menu** → **Analyze Graph** → **Intersection**.

The x-values of the intersections solve $f1(x) = f2(x)$.

EXAMPLE $x^2 = 2x + 3$ gives $x = -1$
and $x = 3$

Domain and range on the GDC: set the window to the domain you need, then read the range from the minimum, maximum or end points.
Give answers to 3 significant figures unless told otherwise, and write down what each point means.

Using quadratic functions to model

A ball is thrown upwards. Its height h metres after t seconds is $h(t) = -5t^2 + 20t$.

EXAMPLE The vertex is at $t = -\frac{20}{2(-5)} = 2$

EXAMPLE $h(2) = -20 + 40 = 20$ so the maximum height is 20 m

When does the ball land?

EXAMPLE $h(t) = 0$

EXAMPLE $-5t(t - 4) = 0$

EXAMPLE $t = 0$ or $t = 4$

EXAMPLE the ball lands after 4 seconds

CHECK 5 What are the domain and range in this problem?

Interpret every answer in context.

Reject answers that do not make sense, such as negative times.

Common mistakes

✗ "The vertex of $y = (x - 3)^2 + 4$ is $(-3, 4)$."

✓ The sign changes: the vertex is $(3, 4)$.

✗ "The axis of symmetry is $y = -\frac{b}{2a}$."

✓ It is a vertical line: $x = -\frac{b}{2a}$.

✗ "The y-intercept is where $y = 0$."

✓ The y-intercept is where $x = 0$. The x-intercepts are where $y = 0$.

✗ "The range of $f(x) = x^2 + 2$ is all real numbers."

✓ The domain is all real numbers. The range is $\{y \in \mathbb{R} : y \geq 2\}$.

Common mistakes (continued)

✗ "If $a > 0$ the vertex is a maximum."

✓ If $a > 0$ the parabola opens upwards, so the vertex is a minimum.

✗ "The x-intercepts of $y = (x - 2)(x + 5)$ are 2 and 5."

✓ They are 2 and -5 . Change the sign inside the bracket.

✗ "With a restricted domain, the range is found from the vertex only."

✓ Also check both end points of the domain.

Answers to the checks

Check 1

Does $f(x) = 3 - 2x^2$ have a maximum or a minimum?

Answer: A maximum, because $a = -2$ is negative.

Check 2

Find the axis of symmetry of $f(x) = x^2 + 8x + 3$

Answer: $x = -\frac{8}{2} = -4$

Check 3

Write $y = (x - 2)(x + 4)$ in standard form

Answer: $x^2 + 4x - 2x - 8 = x^2 + 2x - 8$

Check 4

State the range of $f(x) = x^2 + 2$

Answer: $\{y \in \mathbb{R} : y \geq 2\}$

Check 5

What are the domain and range in this problem?

Answer: Domain $0 \leq t \leq 4$. Range $0 \leq h \leq 20$.