

Year 10 Maths · Unit 10.2 Exponents and Indices

The big idea. Why keep an answer exact when a calculator gives a decimal? Index laws, negative and fractional indices, exponential equations, surds and rationalising denominators.

Key vocabulary

Word	Meaning
Index (power, exponent)	the small number that says how many times to multiply the base, e.g. the 3 in 2^3
Base	the number that is being multiplied by itself, e.g. the 2 in 2^3
Negative index	an index that gives a reciprocal: $2^{-3} = 1/2^3 = 1/8$
Fractional index	an index that means a root: $8^{(1/3)}$ is the cube root of 8
Reciprocal	1 divided by a number: the reciprocal of 8 is $1/8$
Exponential equation	an equation with the unknown in the index, e.g. $2^x = 32$
Rational number	a number that can be written as a fraction of two integers
Irrational number	a number that cannot be written as a fraction, e.g. $\sqrt{2}$
Surd	an irrational root left in exact form, such as $\sqrt{2}$ or $\sqrt{12}$
Like surds	surds with the same root part, e.g. $3\sqrt{2}$ and $5\sqrt{2}$, which can be added or subtracted
Numerator	the number or expression above the line in a fraction
Denominator	the number or expression below the line in a fraction
Rationalise the denominator	rewrite a fraction so that the denominator has no surd
Conjugate	the same two terms with the opposite sign between them: the conjugate of $3 + \sqrt{2}$ is $3 - \sqrt{2}$

Index laws (positive and zero indices)

- $a^m \times a^n = a^{m+n}$ (same base: add the powers). $a^m \div a^n = a^{m-n}$.
- $(a^m)^n = a^{mn}$. $(ab)^n = a^n b^n$.
- $a^0 = 1$ because $a^n \div a^n = a^0 = 1$.
- Expand to see why: $x^3 \times x^2 = (x \cdot x \cdot x)(x \cdot x) = x^5$.
- The laws need the **same base**: $2^3 \times 3^2$ cannot be combined.

Negative and fractional indices

- $a^{-n} = 1 \div a^n$: a negative index is a reciprocal. $2^{-3} = 1/8$.
- $(a/b)^{-n} = (b/a)^n$: flip, then use the positive power.
- $a^{1/n} = \sqrt[n]{a}$. $9^{1/2} = 3$, $27^{1/3} = 3$.

- $a^{m/n} = (\sqrt[n]{a})^m$. $8^{2/3} = (\sqrt[3]{8})^2 = 4$.
- The pattern $2^3 = 8$, $2^2 = 4$, $2^1 = 2$, $2^0 = 1$, $2^{-1} = 1/2$ shows why.

Exponential equations

- If $a^x = a^y$ (same base, $a > 0$, $a \neq 1$) then $x = y$.
- Write both sides with the **same base**: $2^x = 32 = 2^5$, so $x = 5$. $9^x = 27$ gives $3^{2x} = 3^3$ and $x = 3/2$.
- Answers can be negative or fractional: $2^x = 1/8 = 2^{-3}$ gives $x = -3$.
- With an algebraic exponent: $4^{x+1} = 2^{x+7} \rightarrow 2^{2x+2} = 2^{x+7} \rightarrow 2x + 2 = x + 7 \rightarrow x = 5$. Check by substituting.
- The rule fails for base 1 and 0.

Surds

- A **surd** is an irrational root left exact, such as $\sqrt{2}$ or $\sqrt{12}$.
- **Simplify** using the largest square factor: $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$. $\sqrt{50} = 5\sqrt{2}$.
- **Add/subtract** like surds only: $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$. $\sqrt{2} + \sqrt{3}$ stays as it is.
- **Multiply**: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, and $\sqrt{a} \times \sqrt{a} = a$. $2\sqrt{3} \times 3\sqrt{2} = 6\sqrt{6}$.
- **Expand brackets** as normal: $\sqrt{2}(3 + \sqrt{2}) = 3\sqrt{2} + 2$. $(1 + \sqrt{3})^2 = 1 + 2\sqrt{3} + 3 = 4 + 2\sqrt{3}$.

Rationalising the denominator

- **Rationalise** = remove the surd from the denominator, because a rational denominator is easier to use and compare.
- Single surd: $1/\sqrt{2} = (1 \times \sqrt{2})/(\sqrt{2} \times \sqrt{2}) = \sqrt{2}/2$. Multiplying by $\sqrt{2}/\sqrt{2}$ is multiplying by 1, so the value is unchanged.
- **Conjugate**: the same terms with the opposite sign. The conjugate of $3 + \sqrt{2}$ is $3 - \sqrt{2}$.
- $1/(3 + \sqrt{2}) = (3 - \sqrt{2})/((3 + \sqrt{2})(3 - \sqrt{2})) = (3 - \sqrt{2})/(9 - 2) = (3 - \sqrt{2})/7$.
- The product of a number and its conjugate is rational: $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$.

Sequences with surd terms

- Simplify every surd term first: $\sqrt{2}$, $\sqrt{8}$, $\sqrt{18} = \sqrt{2}$, $2\sqrt{2}$, $3\sqrt{2}$.
- That is an arithmetic sequence with $d = \sqrt{2}$.
- Geometric check: $\sqrt{2}$, 2 , $2\sqrt{2}$ has $r = \sqrt{2}$ ($2 \div \sqrt{2} = \sqrt{2}$, and $2\sqrt{2} \div 2 = \sqrt{2}$).
- Write u_n in exact form: $u_n = \sqrt{2} \times n$ for the first sequence.

Common mistakes

- ✗ " $x^3 \times x^2 = x^6$." ✓ Add the powers: x^5 .
- ✗ " $2^{-3} = -8$." ✓ A negative index gives a reciprocal: $1/8$.
- ✗ " $8^{1/3} = 8/3$." ✓ A power of $1/3$ is a cube root: 2.
- ✗ " $\sqrt{2} + \sqrt{3} = \sqrt{5}$." ✓ Unlike surds cannot be added: $\sqrt{2} + \sqrt{3}$ stays as it is.
- ✗ " $(a + b)^2 = a^2 + b^2$." ✓ Expand: $a^2 + 2ab + b^2$. So $(1 + \sqrt{3})^2 = 4 + 2\sqrt{3}$.