

UNIT 10.2

Exponents and Indices

Revision slides: everything you need for the test, one idea per slide.

Key vocabulary (1 of 3)

Index (power, exponent): the small number that says how many times to multiply the base, e.g. the 3 in 2^3

Base: the number that is being multiplied by itself, e.g. the 2 in 2^3

Negative index: an index that gives a reciprocal: $2^{-3} = 1/2^3 = 1/8$

Fractional index: an index that means a root: $8^{(1/3)}$ is the cube root of 8

Reciprocal: 1 divided by a number: the reciprocal of 8 is $1/8$

Key vocabulary (2 of 3)

Exponential equation: an equation with the unknown in the index, e.g. $2^x = 32$

Rational number: a number that can be written as a fraction of two integers

Irrational number: a number that cannot be written as a fraction, e.g. $\sqrt{2}$

Surd: an irrational root left in exact form, such as $\sqrt{2}$ or $\sqrt{12}$

Like surds: surds with the same root part, e.g. $3\sqrt{2}$ and $5\sqrt{2}$, which can be added or subtracted

Key vocabulary (3 of 3)

Numerator: the number or expression above the line in a fraction

Denominator: the number or expression below the line in a fraction

Rationalise the denominator: rewrite a fraction so that the denominator has no surd

Conjugate: the same two terms with the opposite sign between them: the conjugate of $3 + \sqrt{2}$ is $3 - \sqrt{2}$

Index laws (positive and zero indices)

$a^m \times a^n = a^{m+n}$ (same base: add the powers). $a^m \div a^n = a^{m-n}$.

$(a^m)^n = a^{mn}$. $(ab)^n = a^n b^n$.

$a^0 = 1$ because $a^n \div a^n = a^0 = 1$.

Expand to see why: $x^3 \times x^2 = (x \cdot x \cdot x)(x \cdot x) = x^5$.

The laws need the **same base**: $2^3 \times 3^2$ cannot be combined.

Negative and fractional indices

$a^{-n} = 1 \div a^n$: a negative index is a reciprocal. $2^{-3} = 1/8$.

$(a/b)^{-n} = (b/a)^n$: flip, then use the positive power.

$a^{1/n} = \sqrt[n]{a}$. $9^{1/2} = 3$, $27^{1/3} = 3$.

$a^{m/n} = (\sqrt[n]{a})^m$. $8^{2/3} = (\sqrt[3]{8})^2 = 4$.

The pattern $2^3 = 8$, $2^2 = 4$, $2^1 = 2$, $2^0 = 1$, $2^{-1} = 1/2$ shows why.

Exponential equations

If $a^x = a^y$ (same base, $a > 0$, $a \neq 1$) then $x = y$.

Write both sides with the **same base**: $2^x = 32 = 2^5$, so $x = 5$. $9^x = 27$ gives $3^{2x} = 3^3$ and $x = 3/2$.

Answers can be negative or fractional: $2^x = 1/8 = 2^{-3}$ gives $x = -3$.

With an algebraic exponent: $4^{x+1} = 2^{x+7} \rightarrow 2^{2x+2} = 2^{x+7} \rightarrow 2x + 2 = x + 7 \rightarrow x = 5$. Check by substituting.

The rule fails for base 1 and 0.

Surds

A **surd** is an irrational root left exact, such as $\sqrt{2}$ or $\sqrt{12}$.

Simplify using the largest square factor: $\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$. $\sqrt{50} = 5\sqrt{2}$.

Add/subtract like surds only: $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$. $\sqrt{2} + \sqrt{3}$ stays as it is.

Multiply: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, and $\sqrt{a} \times \sqrt{a} = a$. $2\sqrt{3} \times 3\sqrt{2} = 6\sqrt{6}$.

Expand brackets as normal: $\sqrt{2}(3 + \sqrt{2}) = 3\sqrt{2} + 2$. $(1 + \sqrt{3})^2 = 1 + 2\sqrt{3} + 3 = 4 + 2\sqrt{3}$.

Rationalising the denominator

Rationalise = remove the surd from the denominator, because a rational denominator is easier to use and compare.

Single surd: $1/\sqrt{2} = (1 \times \sqrt{2})/(\sqrt{2} \times \sqrt{2}) = \sqrt{2}/2$. Multiplying by $\sqrt{2}/\sqrt{2}$ is multiplying by 1, so the value is unchanged.

Conjugate: the same terms with the opposite sign. The conjugate of $3 + \sqrt{2}$ is $3 - \sqrt{2}$.

$$1/(3 + \sqrt{2}) = (3 - \sqrt{2})/((3 + \sqrt{2})(3 - \sqrt{2})) = (3 - \sqrt{2})/(9 - 2) = (3 - \sqrt{2})/7.$$

The product of a number and its conjugate is rational: $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$.

Sequences with surd terms

Simplify every surd term first: $\sqrt{2}, \sqrt{8}, \sqrt{18} = \sqrt{2}, 2\sqrt{2}, 3\sqrt{2}$.

That is an arithmetic sequence with $d = \sqrt{2}$.

Geometric check: $\sqrt{2}, 2, 2\sqrt{2}$ has $r = \sqrt{2}$ ($2 \div \sqrt{2} = \sqrt{2}$, and $2\sqrt{2} \div 2 = \sqrt{2}$).

Write u_n in exact form: $u_n = \sqrt{2} \times n$ for the first sequence.

Common mistakes

✗ " $x^3 \times x^2 = x^6$."

✓ Add the powers: x^5 .

✗ " $2^{-3} = -8$."

✓ A negative index gives a reciprocal: $1/8$.

✗ " $8^{1/3} = 8/3$."

✓ A power of $1/3$ is a cube root: 2.

✗ " $\sqrt{2} + \sqrt{3} = \sqrt{5}$."

✓ Unlike surds cannot be added: $\sqrt{2} + \sqrt{3}$ stays as it is.

Common mistakes (continued)

✗ " $(a + b)^2 = a^2 + b^2$."

✓ Expand: $a^2 + 2ab + b^2$. So $(1 + \sqrt{3})^2 = 4 + 2\sqrt{3}$.