

Year 10 Maths · Unit 10.1 Sequences

The big idea. How can we be sure a pattern will keep working? Represent sequences, find general rules for arithmetic, geometric and quadratic sequences, and use recursive definitions.

Representing a sequence

- Show a sequence four ways: a **picture**, a **table** of terms, a **graph** (points, not a joined line) and an **algebraic rule**.
- The term number n and the term u_n : u_1 is the first term.
- Triangle numbers: 1, 3, 6, 10, 15 ... rule **$u_n = n(n + 1) \div 2$** . The graph curves upwards.
- A straight-line graph means a constant difference (arithmetic); a curve means something else.

Shifting a known pattern

- Name the base sequence: squares (1, 4, 9, 16), cubes (1, 8, 27, 64), triangle numbers, powers of 2.
- Find the constant shift: 0, 3, 8, 15, 24 is **one less than the squares**, so $u_n = n^2 - 1$.
- Check with a term you did not use: $u_6 = 36 - 1 = 35$.

Quadratic sequences (second differences)

- Build a difference table. If the **first differences** are not constant but the **second differences** are, the rule is quadratic: $u_n = an^2 + bn + c$.
- **$a = \text{second difference} \div 2$** . Subtract an^2 from each term to leave a linear sequence $bn + c$.
- Example 4, 9, 16, 25 ... first differences 5, 7, 9; second differences 2, 2, so $a = 1$; $u_n = n^2 + 2n + 1$.
- Check the rule with a term you did not use.

Arithmetic sequences

- Constant **common difference d** : 5, 8, 11, 14 has $d = 3$.
- **$u_n = u_1 + (n - 1)d$** . For 5, 8, 11: $u_n = 5 + 3(n - 1) = 3n + 2$.
- Find a given term: $u_{20} = 5 + 19 \times 3 = 62$.
- Find which term equals a value: $3n + 2 = 50$ gives $n = 16$.

Geometric sequences

- Constant **common ratio r** : 3, 6, 12, 24 has $r = 2$.
- **$u_n = u_1 \times r^{n-1}$** . For 3, 6, 12: $u_n = 3 \times 2^{n-1}$.
- A negative r alternates the sign: 2, -6, 18, -54 has $r = -3$.
- $u_6 = 3 \times 2^5 = 96$.

Working backwards and equal properties

- Given two terms, count the gap. If $u_3 = 11$ and $u_7 = 27$: 4 steps give 16, so $d = 4$; $u_1 = 11 - 8 = 3$.
- Geometric: if $u_2 = 6$ and $u_5 = 48$, then $r^3 = 8$, $r = 2$ and $u_1 = 3$.
- **Equal differences**: if $x, x + 4, 3x$ are arithmetic then $(x + 4) - x = 3x - (x + 4)$, so $4 = 2x - 4$ and $x = 4$.
- **Equal ratios**: if $a, 6, 24$ are geometric then $6 \div a = 24 \div 6 = 4$, so $a = 1.5$.
- Test each solution by writing out the terms.

Recursive definitions

- A **recursive** definition gives the first term and a rule that uses the previous term.
- Arithmetic: $u_1 = 5$, $u_{n+1} = u_n + 3$. Geometric: $u_1 = 3$, $u_{n+1} = 2u_n$.
- The **explicit** rule gives any term directly: $u_n = 3n + 2$. Convert between them.
- Always state the starting value and use the precise words.

Common mistakes

- ✗ “Join the points of a sequence graph with a line.” ✓ Only whole-number terms exist, so plot separate points.
- ✗ “ $u_n = u_1 + nd$.” ✓ The first term has no difference added: $u_n = u_1 + (n - 1)d$.
- ✗ “Geometric rule: $u_n = u_1 \times r^n$.” ✓ The power is $n - 1$: $u_n = u_1 \times r^{n-1}$.
- ✗ “ a in $an^2 + bn + c$ is the second difference.” ✓ a is half the second difference.
- ✗ “A recursive rule alone is complete.” ✓ You must also state the starting value.