

Practice Booklet

MYP 4 · Year 10 Mathematics (Extended)

Unit 10.1 · Sequences

Name: _____

10.1.1	Representing sequences · Questions 1–15
10.1.2	Shifted sequences · Questions 16–30
10.1.2b	Second differences · Questions 31–45
10.1.3	Arithmetic sequences · Questions 46–60
10.1.4	Geometric sequences · Questions 61–75
10.1.5	Working backwards · Questions 76–90
10.1.6	Equal differences and equal ratios · Questions 91–105
10.1.7	Recursive rules · Questions 106–120
Homework	Homework page and checking record
Answers	For checking and correcting in green pen



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Choosing your level

Every part of the unit has four levels. Start where you feel comfortable and move between them freely. Most questions ask you to explain as well as answer, because being able to say *why* is what makes a method stick.

Learning

Small numbers, with counters or a diagram beside you.

Practising

The standard question, with your reasoning written out.

Challenging

Working backwards, and finding what is wrong with someone else's thinking.

Mastery

Explaining why a rule must be true, and testing it on new cases.

Working on your homework

Pens and setting out

- Answers in blue or black non-erasable pen, or in pencil.
- Colour and highlighters are for annotating your work and showing the steps of a method.
- Never erase a mistake. Rework it underneath so you can see what changed.

Green pen is for checking. When you finish, use the answers at the back to check your work.

- **Got it wrong?** Circle the question number, then make your correction and write your error analysis in green pen.
- **Still can't work out how to solve it?** Put a star ★ next to the question number, so we know to go over it in class.

Each sheet

- Your name goes on every sheet, because they are handed in one at a time.
- Each question is worth up to 3 marks. If a question has parts (a, b, c...), each part is marked out of 3. At the end of each level, write your score out of the total for that level.
- Questions are numbered 1, 2, 3... all the way through the booklet, so we can say exactly which ones to do.
- If you are stuck after trying, ask for help. Showing me what you tried is a good start.

10.1.1 Representing sequences

Name: _____

Key vocabulary

Word	Meaning	Example
sequence	a list of numbers that follows a pattern	1, 3, 6, 10, ...
term	one number in the sequence	the 3rd term is 6
term number, n	the position of a term in the list	$n = 4$ is the 4th term
u_n	the term in position n	$u_4 = 10$
rule	a formula that gives any term from its term number	$u_n = n(n + 1) \div 2$

One sequence can be shown four ways: a **picture**, a **table**, a **graph** and a **rule**.

● Learning

1 The first four terms of a sequence are 1, 3, 6, 10.

- Work out the gaps between the terms.
- Write the next two terms.

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.....

2 A sequence has the rule $u_n = 3n + 1$. Work out u_1 , u_2 and u_3 .

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3 The points (1, 2), (2, 4), (3, 6), (4, 8) are plotted on a graph.

- Write the sequence.
- Do the points lie on a straight line? Explain.
- Write the rule $u_n = \underline{\hspace{2cm}}$

.....

.....

4 Staircase pictures: step 1 has 1 square, step 2 has 4 squares (a 2 by 2 block), step 3 has 9 squares. Draw step 4, count its squares and write the rule.

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Learning score _____ / 21

Parent / guardian signature: _____



10.1.1 Representing sequences

PRACTISING

Name: _____

● Practising

5 The triangle numbers have the rule $u_n = \frac{n(n+1)}{2}$. Work out

a) u_8

b) u_{10}

.....

.....

6 Complete the table for $u_n = n^2 + 3$ with $n = 1$ to 5.

a) Complete the table.

b) Do the points lie on a straight line? Explain.

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.....

7 A sequence goes 5, 8, 11, 14, 17.

a) Find the gaps.

b) Will the graph be a straight line or a curve? Explain.

c) Write a rule for u_n .

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8 Is 45 a triangle number? Solve $n(n+1) = 2 \times 45$ by looking for two numbers next to each other with product 90.

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Practising score _____ / 24

Parent / guardian signature: _____



10.1.1 Representing sequences

CHALLENGING · MASTERY

Name: _____

● Challenging

- 9 Compare 2, 6, 12, 20, 30 with the triangle numbers 1, 3, 6, 10, 15. Write the rule for u_n .

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- 10 Find and describe the mistake. Dev says the sequence 1, 3, 6, 10, 15 has the rule $u_n = 2n - 1$ because $u_1 = 1$ and $u_2 = 3$.

.....

- 11 Step 11 of the triangle pattern has 66 counters and step 12 has 78. How many counters are added, and why does that number make sense in the picture?

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- 12 Is 100 a triangle number? Explain using $n(n + 1) = 200$.

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Challenging score _____ / 12

● Mastery

- 13 Draw two copies of step 5 of the triangle pattern fitted together to make a rectangle. Use it to explain why $u_5 = \frac{5 \times 6}{2}$.

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- 14 Sam says that a table of five terms proves a rule. Explain why it does not, and say how the rectangle picture does prove the rule for every n .

.....

- 15 A graph of a sequence has points that curve upwards. Write the first five terms of one possible sequence and explain how you know the graph curves without plotting it.

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Mastery score _____ / 9

Parent / guardian signature: _____



10.1.2 Shifted sequences

LEARNING

Name: _____

Name it, shift it, check it

Step	What you do	Example: 2, 5, 10, 17, 26
Name it	Which known sequence is it close to? Squares or cubes	1, 4, 9, 16, 25
Shift it	Subtract. What constant is added or taken away every time?	always + 1
Check it	Substitute a term you did not use	$n = 5: 25 + 1 = 26$

Squares: 1, 4, 9, 16, 25. Cubes: 1, 8, 27, 64, 125.

● Learning

16 Write the first five square numbers and the first five cube numbers.

.....

.....

17 Compare 0, 3, 8, 15, 24 with the squares 1, 4, 9, 16, 25.

a) What is added to each square to give the term?

b) Complete the rule $u_n = n^2$ ____

.....

.....

18 A sequence has the rule $u_n = n^2 + 2$. Work out u_1 , u_2 , u_3 and u_4 .

.....

.....

19 Name it, shift it: find the rule for 2, 5, 10, 17, 26.

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Learning score _____ / 15

Parent / guardian signature: _____

10.1.2 Shifted sequences

PRACTISING

Name: _____

● Practising

20 Find the rule for

a) 5, 8, 13, 20, 29

b) -1, 2, 7, 14, 23

.....

.....

21 Find the rule for

a) 0, 7, 26, 63, 124

b) 2, 9, 28, 65, 126 (compare with the cubes)

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.....

22 A sequence has the rule $u_n = n^2 - 3$.

a) Find u_8 .

b) Which term equals 97?

c) Is 45 a term? Explain.

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23 The terms 3, 6, 11, 18 were used to find the rule $n^2 + 2$. Check the rule with the fifth term, 27.

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Practising score _____ / 24

Parent / guardian signature: _____



10.1.2 Shifted sequences

CHALLENGING · MASTERY

Name: _____

● Challenging

24 Find the rule for 3, 10, 29, 66, 127.

25 Find and describe the mistake. Ali says the rule for 4, 7, 12, 19 is $n^2 + 4$ because the first term is 4.

26 a) Find the rule for -4, -1, 4, 11, 20. b) Which term equals 76?

27 The sequence 10, 17, 36, 73, 134 is the cubes shifted by a constant. Write the rule and find u_{10} .

Challenging score _____ / 15

● Mastery

28 Why is it important to test a term you did not use when you build a rule? Explain.

29 How can you tell from the terms of a sequence whether to compare it with the squares or with the cubes? Use two examples.

30 Make your own sequence by shifting the squares by a number you choose. Write the first five terms and the rule, then swap with a partner to check.

Mastery score _____ / 9

Parent / guardian signature: _____



10.1.2b Second differences

LEARNING

Name: _____

Key vocabulary

Word	Meaning	Example: 2, 6, 12, 20, 30
first difference	the gap between one term and the next	4, 6, 8, 10
second difference	the gap between one first difference and the next	2, 2, 2
quadratic sequence	a sequence whose second differences are constant	rule $an^2 + bn + c$

a is half of the second difference. Then $b = (\text{first gap}) - 3a$, and $c = u_1 - a - b$.

● Learning

31 Find the first differences and the second differences of 2, 6, 12, 20, 30.

.....

.....

32 Find the first and second differences of 1, 4, 9, 16, 25. Is the sequence quadratic?

.....

.....

33 Each sequence has a constant second difference. Find a , which is half of the second difference.

- a) second difference 2
- b) second difference 6
- c) second difference 10

.....

.....

34 For $u_n = n^2 + 2n$ find u_1 to u_4 . Then check that the second differences are constant.

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.....

Learning score _____ / 18

Parent / guardian signature: _____



10.1.2b Second differences

PRACTISING

Name: _____

● Practising

35 Find the rule for

a) 6, 9, 14, 21

b) 2, 7, 14, 23

.....

.....

36 Find the rule for 3, 10, 21, 36, 55.

.....

.....

37 Find the rule for 5, 4, 5, 8, 13.

.....

.....

38 A sequence has the rule $u_n = 2n^2 - 3n$.

a) Find u_{10} .

b) Which term equals 20? Try values of n .

.....

.....

Practising score _____ / 18

Parent / guardian signature: _____



10.1.2b Second differences

CHALLENGING · MASTERY

Name: _____

● Challenging

39 Find the rule for 7, 18, 33, 52, 75.

40 Find and describe the mistake. Mia says that the second difference of 3, 8, 15, 24 is 2, so $a = 2$.

41 The sequence 5, 11, 21, 35, 53 is quadratic. Find the rule, then find u_{20} .

42 The first differences of a quadratic sequence start 5, 9, 13 and its first term is 3. Write the first four terms and find the rule.

Challenging score _____ / 12

● Mastery

43 Explain why the second difference of $an^2 + bn + c$ is $2a$. Use the terms for $n = 1, 2, 3$.

44 Explain why 2, 4, 8, 16, 32 is not a quadratic sequence.

45 A sequence has the rule $u_n = n^2 + bn$ and $u_2 = 10$. Find b and write the first four terms.

Mastery score _____ / 9

Parent / guardian signature: _____



10.1.3 Arithmetic sequences

LEARNING

Name: _____

Key vocabulary

Word	Meaning	Example: 4, 9, 14, 19
arithmetic sequence	the same amount is added every time	+ 5 each time
common difference, d	the amount added (negative if the terms fall)	$d = 5$
u_1	the first term	$u_1 = 4$
rule	$u_n = u_1 + (n - 1)d$	$4 + (n - 1) \times 5 = 5n - 1$

● Learning

46 Find d for

a) 4, 9, 14, 19

b) 50, 42, 34, 26

.....

47 An arithmetic sequence has $u_1 = 6$ and $d = 4$. Write u_2 to u_5 .

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48 Use $u_n = u_1 + (n - 1)d$ to write the rule for 3, 7, 11, 15.

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49 An arithmetic sequence has the rule $u_n = 5n - 2$. Write

a) u_1

b) d

c) u_{10}

.....

.....

.....

Learning score _____ / 21

Parent / guardian signature: _____



10.1.3 Arithmetic sequences

PRACTISING

Name: _____

● Practising

50 Find the rule for 8, 15, 22, 29 and use it to find u_{40} .

.....

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51 Find the rule for 90, 84, 78, 72 and use it to find u_{18} .

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52 An arithmetic sequence has $u_1 = 1.5$ and $d = 0.25$. Find u_{12} .

.....

.....

53 Which term of 5, 8, 11, 14, ... equals 101?

.....

.....

Practising score _____ / 12

Parent / guardian signature: _____



10.1.3 Arithmetic sequences

CHALLENGING · MASTERY

Name: _____

● Challenging

54 An arithmetic sequence has $u_1 = -4$ and $d = -1.5$. Find u_n and u_9 .

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55 Find and describe the mistake. Ben says u_{30} of 4, 9, 14, ... is $4 + 30 \times 5 = 154$.

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56 An arithmetic sequence has $u_1 = 100$ and $d = -7$. Find the first negative term.

.....

57 Is 250 a term of 7, 12, 17, ...? Explain.

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.....

Challenging score _____ / 12

● Mastery

58 Use the jumps on a number line to explain why $u_n = u_1 + (n - 1)d$ and not $u_1 + nd$.

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59 A sequence has $d = 0$. What does its graph look like? Is it still an arithmetic sequence?

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60 Show that $u_n = 3n + 4$ is arithmetic by working out $u_{n+1} - u_n$.

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Mastery score _____ / 9

Parent / guardian signature: _____

10.1.4 Geometric sequences

LEARNING

Name: _____

Key vocabulary

Word	Meaning	Example: 3, 6, 12, 24
geometric sequence	each term is multiplied by the same number	$\times 2$ each time
common ratio, r	the multiplier (divide a term by the one before)	$r = 2$
rule	$u_n = u_1 \times r^{n-1}$	$3 \times 2^{n-1}$

● Learning

61 Find r for

a) 2, 6, 18, 54

b) 400, 200, 100, 50

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62 A geometric sequence has $u_1 = 4$ and $r = 3$. Write u_2 to u_5 .

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63 Use $u_n = u_1 \times r^{n-1}$ to write the rule for 5, 10, 20, 40.

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.....

64 A geometric sequence has $u_1 = 3$ and $r = 2$. Write u_1 to u_6 .

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.....

Learning score _____ / 15

Parent / guardian signature: _____



10.1.4 Geometric sequences

PRACTISING

Name: _____

● Practising

65 Find the rule for 3, 9, 27, 81 and use it to find u_8 .

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.....

66 Find the rule for 1000, 500, 250, 125 and use it to find u_7 .

.....
.....

67 A geometric sequence has $u_1 = -2$ and $r = 5$. Find u_4 .

.....
.....

68 Which term of 2, 6, 18, 54, ... equals 1458?

.....
.....

Practising score _____ / 12

Parent / guardian signature: _____



10.1.4 Geometric sequences

CHALLENGING · MASTERY

Name: _____

● Challenging

69 A geometric sequence has $u_1 = 6$ and $r = -\frac{1}{2}$. Find u_n and u_6 .

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.....

70 Find and describe the mistake. Zoe says u_5 of 2, 6, 18, ... is $2 \times 3^5 = 486$.

.....
.....

71 Find the rule for 1, -2, 4, -8, ... and say, without listing every term, whether u_{15} is positive or negative.

.....
.....

72 £500 is invested and grows by 4% each year. The amounts form a geometric sequence with $u_1 = 500$ at the start. Write r and find the amount after 3 years.

.....
.....

Challenging score _____ / 12

● Mastery

73 Use the doubling bars picture to explain why $u_n = u_1 \times r^{n-1}$ and not $u_1 \times r^n$.

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74 A sequence has $r = 1$. What does its graph look like? Is it also an arithmetic sequence? Explain.

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75 A geometric sequence has $0 < r < 1$ and $u_1 > 0$. Explain why the terms get smaller but never reach 0.

.....
.....

Mastery score _____ / 9

Parent / guardian signature: _____



10.1.5 Working backwards

LEARNING · PRACTISING

Name: _____

Working backwards

Sequence Jump between two terms

Sequence	Jump between two terms	Example
arithmetic	from u_a to u_b there are $(b - a)$ jumps, so $d = (\text{difference of terms}) \div (b - a)$	$u_3 = 8, u_6 = 23: d = 15 \div 3 = 5$
geometric	$r^{b-a} = u_b \div u_a$, then take the root	$u_2 = 6, u_5 = 162: r^3 = 27$

● Learning

76 An arithmetic sequence has $u_1 = 6$ and $u_2 = 11$. Find d and u_3 .

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77 A geometric sequence has $u_1 = 5$ and $u_2 = 15$. Find r and u_3 .

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78 An arithmetic sequence has $u_3 = 14$ and $d = 4$. Work back to find u_1 (subtract d twice).

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79 A geometric sequence has $u_3 = 36$ and $r = 3$. Work back to find u_1 (divide by r twice).

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Learning score _____ / 12

● Practising

80 An arithmetic sequence has $u_3 = 8$ and $u_6 = 23$. Find d and u_1 .

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81 A geometric sequence has $u_2 = 6$ and $u_5 = 162$. Find r and u_1 .

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82 An arithmetic sequence has the rule $u_n = 7n - 4$. Which term equals 87?

.....

.....

83 A geometric sequence has the rule $u_n = 2 \times 3^{n-1}$. Which term equals 486?

.....

.....

Practising score _____ / 12

Parent / guardian signature: _____

10.1.5 Working backwards

CHALLENGING · MASTERY

Name: _____

● Challenging

84 A geometric sequence has $u_4 = 3$ and $u_7 = 6$. Find r exactly, without rounding.

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85 An arithmetic sequence has $u_2 = 15$ and $u_5 = -3$. Find d and u_1 .

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86 Find and describe the mistake. Kai says an arithmetic sequence with $u_2 = 7$ and $u_6 = 23$ has $d = (23 - 7) \div 6$.

.....

.....

87 An arithmetic sequence has $u_1 = 5$, $d = 4$ and $u_n = 77$. How many terms are there up to u_n ?

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Challenging score _____ / 12

● Mastery

88 A geometric sequence has $u_2 = 4$ and $u_6 = 64$. Find both possible sequences and their u_1 .

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89 Explain why $u_6 \div u_2 = r^4$ for a geometric sequence.

.....

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90 An arithmetic sequence has $u_4 = 10$ and $u_9 = -5$. Is 0 a term? Explain.

.....

.....

Mastery score _____ / 9

Parent / guardian signature: _____

10.1.6 Equal differences and equal ratios

Name: _____

The two properties

Three consecutive terms a, b, c

arithmetic

geometric

What is equal

$b - a = c - b$ (equal differences)

$b \div a = c \div b$ (equal ratios)

So

b is the mean of a and c

$b^2 = ac$

● Learning

91 $x, x + 5, x + 10$ are consecutive terms of an arithmetic sequence. Show that both differences equal 5 for any x .

.....

.....

92 4, 8, m are consecutive terms of a geometric sequence. Find m .

.....

.....

93 3, k , 11 are consecutive terms of an arithmetic sequence. Find k .

.....

.....

94 2, k , 18 are consecutive terms of a geometric sequence with $k > 0$. Find k .

.....

.....

Learning score _____ / 12

Parent / guardian signature: _____



10.1.6 Equal differences and equal ratios

PRACTISING

Name: _____

● Practising

- 95** n , $3n - 2$, $4n + 1$ are consecutive terms of an arithmetic sequence. Find n and the three terms.

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- 96** 5, k , 45 are consecutive terms of a geometric sequence. Find both possible values of k and the sequence each gives.

.....

.....

- 97** $x + 2$, $2x + 1$, $4x - 3$ are consecutive terms of an arithmetic sequence. Find x and the terms.

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- 98** x , $x + 4$, $x + 10$ are consecutive terms of a geometric sequence. Find x and the common ratio.

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Practising score _____ / 12

Parent / guardian signature: _____



10.1.6 Equal differences and equal ratios

CHALLENGING · MASTERY

Name: _____

● Challenging

99 $n + 1$, $2n + 3$, $4n$ are consecutive terms of an arithmetic sequence. Find n and the terms.

.....

100 Find and describe the mistake. To find k in 3, k , 27 (geometric), Ravi writes $k - 3 = 27 - k$, so $k = 15$.

.....

101 $x - 1$, $x + 1$, $2x + 2$ are consecutive terms of a geometric sequence. Show that $x^2 - 2x - 3 = 0$, solve it, and say which value gives a valid geometric sequence.

.....

102 Three numbers in an arithmetic sequence add to 24 and the largest is 3 times the smallest. Find them.

.....

Challenging score _____ / 12

● Mastery

103 Explain why the middle term of three consecutive terms of an arithmetic sequence is the mean of the other two.

.....

104 Explain why, for three positive consecutive terms of a geometric sequence, the middle term is the square root of the product of the other two.

.....

105 Can -5 , k , -20 be consecutive terms of a geometric sequence? Find all possible values of k .

.....

Mastery score _____ / 9

Parent / guardian signature: _____



10.1.7 Recursive rules

LEARNING

Name: _____

Key vocabulary

Word	Meaning	Example
recursive rule	gives the first term and how to get each term from the one before	$u_1 = 5, u_{n+1} = u_n + 3$
explicit rule	gives any term straight from n	$u_n = 3n + 2$

Arithmetic: $u_{n+1} = u_n + d$. Geometric: $u_{n+1} = r \times u_n$. Always state u_1 .

● Learning

106 A sequence is defined by $u_1 = 5$ and $u_{n+1} = u_n + 3$. Write u_2 , u_3 and u_4 .

.....

.....

107 A sequence is defined by $u_1 = 2$ and $u_{n+1} = 3u_n$. Write u_2 , u_3 and u_4 .

.....

.....

108 Say in words what $u_{n+1} = u_n - 4$ means.

.....

.....

109 The sequence 20, 17, 14, 11 is arithmetic. Write its recursive rule.

.....

.....

Learning score _____ / 12

Parent / guardian signature: _____

10.1.7 Recursive rules

PRACTISING

Name: _____

● Practising

110 A sequence has $u_1 = 4$ and $u_{n+1} = u_n + 7$. Find u_2 to u_5 , then write the explicit rule.

.....

111 A sequence has $u_1 = 100$ and $u_{n+1} = 0.5u_n$. Find u_2 to u_5 , then write the explicit rule.

.....

112 Write a recursive rule for

a) $u_n = 8 - 2n$

b) $u_n = 3 \times 2^{n-1}$

.....

113 A sequence has $u_1 = 6$ and $u_{n+1} = -3u_n$. Write the explicit rule.

.....

.....

Practising score _____ / 15

Parent / guardian signature: _____



10.1.7 Recursive rules

CHALLENGING · MASTERY

Name: _____

● Challenging

114 A sequence has $u_1 = 2$ and $u_{n+1} = 2u_n + 1$. Find u_2 to u_5 . Is the sequence arithmetic, geometric or neither? Explain.

.....

.....

115 Find and describe the mistake. Tom writes the sequence 5, 9, 13, 17 as $u_1 = 5$ and $u_{n+1} = 4u_n$.

.....

.....

116 A sequence has $u_1 = 80$ and $u_{n+1} = 1.1u_n$. Find the first term that is greater than 100.

.....

.....

117 An arithmetic sequence has $u_1 = 7$ and $u_4 = 22$. Write its recursive rule and its explicit rule.

.....

.....

Challenging score _____ / 12

● Mastery

118 A sequence has $u_1 = 3$ and $u_{n+1} = u_n + 2n$. Find u_2 to u_5 and explain why no single d fits.

.....

.....

119 Give one advantage of an explicit rule and one advantage of a recursive rule.

.....

.....

120 For 5, 10, 20, 40, ... write the recursive and the explicit rule. Which is quicker for finding u_{30} , and why?

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.....

Mastery score _____ / 9

Parent / guardian signature: _____

Name: _____

Your teacher will tell you which sheet and questions to do. Choose your level, set out your work in pen or pencil, then check it with the answers at the back using a **green pen**: circle wrong question numbers, and star ★ any you still cannot solve.

Sheet	Questions	Which ones I did	Checked in green ✓	Teacher
10.1.1 Representing sequences	Q1–15			
10.1.2 Shifted sequences	Q16–30			
10.1.2b Second differences	Q31–45			
10.1.3 Arithmetic sequences	Q46–60			
10.1.4 Geometric sequences	Q61–75			
10.1.5 Working backwards	Q76–90			
10.1.6 Equal differences and equal ratios	Q91–105			
10.1.7 Recursive rules	Q106–120			

Error analysis

Circle the number of any question you got wrong. Star ★ it if you still cannot solve it after correcting. Then write the questions you need to rework, and what you will do differently:

One thing I understand better now

Score: _____

Parent / guardian signature: _____

10.1.1 Representing sequences

Learning	1	a) 2, 3, 4 (the gaps grow by 1 each time) b) 15 and 21
	2	4, 7, 10 (put $n = 1, 2, 3$ into the rule)
	3	a) 2, 4, 6, 8 b) Yes, the same amount (2) is added each time c) $u_n = 2n$
	4	Step 4 is a 4 by 4 block, 16 squares. Rule: $u_n = n^2$
Practising	5	a) $\frac{8 \times 9}{2} = 36$ b) $\frac{10 \times 11}{2} = 55$
	6	a) 4, 7, 12, 19, 28 b) No. The gaps 3, 5, 7, 9 keep growing, so the graph curves upwards
	7	a) 3, 3, 3, 3 b) A straight line, because the gap is constant c) $u_n = 3n + 2$
	8	$90 = 9 \times 10$, so $n = 9$. Yes, 45 is the 9th triangle number: $\frac{9 \times 10}{2} = 45$
Challenging	9	Each term is double the triangle number, so $u_n = 2 \times \frac{n(n+1)}{2} = n(n+1)$
	10	The rule only works for the first two terms. $2n - 1$ gives $u_3 = 5$, but the third term is 6. A rule must work for every term, so always test a term you did not use
	11	12 counters. Step 12 is step 11 with a new bottom row of 12 counters
	12	No. $13 \times 14 = 182$ and $14 \times 15 = 210$, so no two whole numbers next to each other multiply to 200
Mastery	13	The two copies make a rectangle with 5 rows and 6 columns, 30 squares. One staircase is half of that, which is 15
	14	A table only shows the rule works for the terms you tested. The picture works for any n : step n is half of a rectangle with n rows and $(n + 1)$ columns
	15	For example 1, 4, 9, 16, 25. The gaps 3, 5, 7, 9 are not constant, so the points do not lie on a straight line and the graph curves

10.1.2 Shifted sequences

Learning	16	Squares: 1, 4, 9, 16, 25. Cubes: 1, 8, 27, 64, 125
	17	a) -1 (each term is 1 less than the square) b) $u_n = n^2 - 1$
	18	3, 6, 11, 18
	19	$u_n = n^2 + 1$ (each term is the square plus 1)
Practising	20	a) $u_n = n^2 + 4$ b) $u_n = n^2 - 2$
	21	a) $u_n = n^3 - 1$ b) $u_n = n^3 + 1$
	22	a) $64 - 3 = 61$ b) $n^2 = 100$, so $n = 10$ c) No. $n^2 = 48$ and 48 is not a square number
	23	$n = 5$: $25 + 2 = 27$. The rule works for a term that was not used to build it
Challenging	24	$u_n = n^3 + 2$ (cubes 1, 8, 27, 64, 125, each plus 2)
	25	$n = 1$ gives $1 + 4 = 5$, not 4. The first term is 4, which is 3 more than 1, so the shift is +3. The rule is $n^2 + 3$
	26	a) $u_n = n^2 - 5$ b) $n^2 - 5 = 76$, so $n^2 = 81$ and $n = 9$
	27	$u_n = n^3 + 9$; $u_{10} = 1000 + 9 = 1009$
Mastery	28	The rule was built from the terms you used, so it must work for them. A different term is the first real test that the rule works for the whole sequence
	29	Look at how fast it grows. 2, 5, 10, 17 grows slowly (gaps 3, 5, 7), like squares. 2, 9, 28, 65 grows fast (gaps 7, 19, 37), like cubes
	30	For example, shift by +6: 7, 10, 15, 22, 31 with rule $n^2 + 6$. Any rule $n^2 + k$ works if the terms match

10.1.2b Second differences

Learning	31	First differences: 4, 6, 8, 10. Second differences: 2, 2, 2
	32	First: 3, 5, 7, 9. Second: 2, 2, 2. The second differences are constant, so it is quadratic
	33	a) $a = 1$ b) $a = 3$ c) $a = 5$
	34	3, 8, 15, 24. First differences 5, 7, 9; second differences 2, 2
Practising	35	a) gaps 3, 5, 7, second difference 2, $a = 1$, $b = 0$, $c = 5$: $n^2 + 5$ b) gaps 5, 7, 9, $a = 1$, $b = 5 - 3 = 2$, $c = 2 - 1 - 2 = -1$: $n^2 + 2n - 1$
	36	Gaps 7, 11, 15, 19, second difference 4, $a = 2$, $b = 7 - 6 = 1$, $c = 3 - 2 - 1 = 0$: $u_n = 2n^2 + n$
	37	Gaps -1, 1, 3, 5, second difference 2, $a = 1$, $b = -1 - 3 = -4$, $c = 5 - 1 + 4 = 8$: $u_n = n^2 - 4n + 8$
	38	a) $200 - 30 = 170$ b) $n = 4$: $32 - 12 = 20$, so it is the 4th term
Challenging	39	Gaps 11, 15, 19, 23, second difference 4, $a = 2$, $b = 11 - 6 = 5$, $c = 7 - 2 - 5 = 0$: $u_n = 2n^2 + 5n$
	40	a is half of the second difference, so $a = 1$. Check: the rule is $n^2 + 2n$, which gives 3, 8, 15, 24
	41	Gaps 6, 10, 14, 18, second difference 4, $a = 2$, $b = 6 - 6 = 0$, $c = 5 - 2 = 3$: $u_n = 2n^2 + 3$. $u_{20} = 800 + 3 = 803$
	42	3, 8, 17, 30. Second difference 4, $a = 2$, $b = 5 - 6 = -1$, $c = 3 - 2 + 1 = 2$: $u_n = 2n^2 - n + 2$
Mastery	43	The terms are $a + b + c$, $4a + 2b + c$, $9a + 3b + c$. The first differences are $3a + b$ and $5a + b$. The second difference is $(5a + b) - (3a + b) = 2a$
	44	First differences 2, 4, 8, 16 and second differences 2, 4, 8 are not constant, so it is not quadratic (it doubles each time, so it is geometric)
	45	$4 + 2b = 10$, so $b = 3$. The terms are 4, 10, 18, 28

10.1.3 Arithmetic sequences

Learning	46	a) $d = 5$ b) $d = -8$ (the terms are falling)
	47	10, 14, 18, 22
	48	$u_1 = 3$ and $d = 4$, so $u_n = 3 + 4(n - 1) = 4n - 1$
	49	a) 3 b) 5 c) 48
Practising	50	$u_n = 8 + 7(n - 1) = 7n + 1$. $u_{40} = 281$
	51	$u_n = 90 - 6(n - 1) = 96 - 6n$. $u_{18} = -12$
	52	$u_{12} = 1.5 + 11 \times 0.25 = 4.25$
	53	$u_n = 3n + 2$. $3n + 2 = 101$, so $n = 33$. It is the 33rd term
Challenging	54	$u_n = -4 - 1.5(n - 1) = -2.5 - 1.5n$. $u_9 = -16$
	55	There are only 29 jumps from u_1 to u_{30} , not 30. $u_{30} = 4 + 29 \times 5 = 149$
	56	$100 - 7(n - 1) < 0$ gives $n - 1 > 14.3$, so $n = 16$. $u_{15} = 2$ and $u_{16} = -5$, so the first negative term is $u_{16} = -5$
	57	$u_n = 5n + 2$. $5n + 2 = 250$ gives $n = 49.6$, which is not a whole number, so 250 is not a term
Mastery	58	You start on u_1 with no jumps. To reach the n th term you make $n - 1$ jumps of size d , so the jumps total $(n - 1)d$
	59	The points lie on a horizontal line. Yes, because the same amount (0) is added every time
	60	$u_{n+1} = 3(n + 1) + 4 = 3n + 7$, so $u_{n+1} - u_n = 3$. The difference is the constant 3, so it is arithmetic with $d = 3$

10.1.4 Geometric sequences

Learning	61	a) $r = 3$ b) $r = \frac{1}{2}$ (or 0.5)
	62	12, 36, 108, 324
	63	$u_1 = 5$ and $r = 2$, so $u_n = 5 \times 2^{n-1}$
	64	3, 6, 12, 24, 48, 96
Practising	65	$u_n = 3 \times 3^{n-1} = 3^n$. $u_8 = 3^8 = 6561$
	66	$u_n = 1000 \times \frac{1}{2}^{n-1}$. $u_7 = 1000 \times \frac{1}{64} = 15.625$
	67	$u_4 = -2 \times 5^3 = -250$
	68	$2 \times 3^{n-1} = 1458$, so $3^{n-1} = 729 = 3^6$. $n - 1 = 6$, so $n = 7$
Challenging	69	$u_n = 6 \times (-\frac{1}{2})^{n-1}$. $u_6 = 6 \times (-\frac{1}{32}) = -\frac{3}{16} = -0.1875$
	70	The power is $n - 1$, not n . There are 4 multiplications from u_1 to u_5 . $u_5 = 2 \times 3^4 = 162$
	71	$u_n = (-2)^{n-1}$. For $n = 15$ the power is 14, which is even, so u_{15} is positive ($u_{15} = 16\,384$)
	72	$r = 1.04$. After 3 years it is $u_4 = 500 \times 1.04^3 = \text{£}562.43$ (to the nearest penny)
Mastery	73	The first bar is u_1 before any multiplying. Each later bar is multiplied by r once more, so the n th bar has been multiplied by r exactly $n - 1$ times
	74	The points lie on a horizontal line. Yes. The terms are all equal, so it is arithmetic with $d = 0$ as well
	75	Each term is multiplied by a number between 0 and 1, so it becomes a smaller fraction of itself. A positive number multiplied by a positive number is never 0 and never negative

10.1.5 Working backwards

Learning	76	$d = 5$ and $u_3 = 16$
	77	$r = 15 \div 5 = 3$ and $u_3 = 45$
	78	$u_2 = 10$ and $u_1 = 6$
	79	$u_2 = 12$ and $u_1 = 4$
Practising	80	3 jumps from u_3 to u_6 : $d = 15 \div 3 = 5$. $u_1 = 8 - 2 \times 5 = -2$
	81	$r^3 = 162 \div 6 = 27$, so $r = 3$. $u_1 = 6 \div 3 = 2$
	82	$7n - 4 = 87$, so $7n = 91$ and $n = 13$
	83	$3^{n-1} = 243 = 3^5$, so $n - 1 = 5$ and $n = 6$
Challenging	84	$r^3 = 6 \div 3 = 2$, so $r = \sqrt[3]{2}$ (about 1.26)
	85	$d = (-3 - 15) \div 3 = -6$ and $u_1 = 15 + 6 = 21$
	86	There are 4 jumps from u_2 to u_6 , not 6. $d = 16 \div 4 = 4$ and $u_1 = 3$
	87	$5 + 4(n - 1) = 77$, so $n - 1 = 18$ and $n = 19$
Mastery	88	$r^4 = 16$, so $r = 2$ or $r = -2$. If $r = 2$ then $u_1 = 2$. If $r = -2$ then $u_1 = -2$. Both give $u_2 = 4$ and $u_6 = 64$
	89	From u_2 to u_6 the term is multiplied by r four times, so $u_6 = u_2 \times r^4$
	90	$d = -15 \div 5 = -3$ and $u_1 = 19$. $19 - 3(n - 1) = 0$ gives $n - 1 = 6.33$, not a whole number, so 0 is not a term

10.1.6 Equal differences and equal ratios

Learning	91	$(x + 5) - x = 5$ and $(x + 10) - (x + 5) = 5$. The x cancels, so the differences are 5 for any x
	92	$r = 8 \div 4 = 2$, so $m = 16$
	93	$k - 3 = 11 - k$, so $2k = 14$ and $k = 7$
	94	$k \div 2 = 18 \div k$, so $k^2 = 36$ and $k = 6$
Practising	95	$(3n - 2) - n = (4n + 1) - (3n - 2)$, so $2n - 2 = n + 3$ and $n = 5$. The terms are 5, 13, 21
	96	$k^2 = 225$, so $k = 15$ or $k = -15$. The sequences are 5, 15, 45 and 5, -15, 45
	97	$(2x + 1) - (x + 2) = (4x - 3) - (2x + 1)$, so $x - 1 = 2x - 4$ and $x = 3$. The terms are 5, 7, 9
	98	$(x + 4)^2 = x(x + 10)$, so $x^2 + 8x + 16 = x^2 + 10x$ and $x = 8$. The terms are 8, 12, 18 and $r = 1.5$
Challenging	99	$(2n + 3) - (n + 1) = 4n - (2n + 3)$, so $n + 2 = 2n - 3$ and $n = 5$. The terms are 6, 13, 20
	100	He used equal differences, which is the arithmetic property. For a geometric sequence the ratios are equal: $k \div 3 = 27 \div k$, so $k^2 = 81$ and $k = 9$ or -9
	101	$(x + 1)^2 = (x - 1)(2x + 2)$, so $x^2 + 2x + 1 = 2x^2 - 2$, which gives $x^2 - 2x - 3 = 0$ and $x = 3$ or $x = -1$. $x = 3$ gives 2, 4, 8 ($r = 2$). $x = -1$ gives -2, 0, 0, which is not geometric (the ratio is not defined)
	102	The middle term is $24 \div 3 = 8$. Write $8 - d, 8, 8 + d$. $8 + d = 3(8 - d)$, so $4d = 16$ and $d = 4$. The numbers are 4, 8, 12
Mastery	103	$b - a = c - b$ gives $2b = a + c$, so $b = \frac{a + c}{2}$, which is the mean of a and c
	104	$b \div a = c \div b$ gives $b^2 = ac$, so $b = \sqrt{ac}$ when b is positive
	105	$k^2 = (-5)(-20) = 100$, so $k = 10$ or $k = -10$. Both work: -5, 10, -20 has $r = -2$ and -5, -10, -20 has $r = 2$

10.1.7 Recursive rules

Learning	106	8, 11, 14
	107	6, 18, 54
	108	Each term is the term before it minus 4 (the sequence goes down by 4 each time)
	109	$u_1 = 20$ and $u_{n+1} = u_n - 3$
Practising	110	11, 18, 25, 32. $u_n = 4 + 7(n - 1) = 7n - 3$
	111	50, 25, 12.5, 6.25. $u_n = 100 \times 0.5^{n-1}$
	112	a) $u_1 = 6$ and $u_{n+1} = u_n - 2$ b) $u_1 = 3$ and $u_{n+1} = 2u_n$
	113	$u_n = 6 \times (-3)^{n-1}$
Challenging	114	5, 11, 23, 47. The differences 3, 6, 12 are not constant and the ratios 2.5, 2.2, 2.09... are not constant, so it is neither
	115	$4u_n$ multiplies each term by 4, which gives 5, 20, 80. The sequence adds 4 each time, so the rule is $u_{n+1} = u_n + 4$
	116	$u_2 = 88$, $u_3 = 96.8$, $u_4 = 106.48$. The first term greater than 100 is u_4
	117	$d = 15 \div 3 = 5$. Recursive: $u_1 = 7$ and $u_{n+1} = u_n + 5$. Explicit: $u_n = 5n + 2$
Mastery	118	5, 9, 15, 23. The amounts added are 2, 4, 6, 8, so the difference changes and there is no constant d
	119	Explicit: you can find any term, such as u_{100} , directly. Recursive: it shows how the sequence grows from term to term and is easy to enter in a calculator
	120	Recursive: $u_1 = 5$ and $u_{n+1} = 2u_n$. Explicit: $u_n = 5 \times 2^{n-1}$. The explicit rule is quicker: $u_{30} = 5 \times 2^{29}$ in one step, instead of 29 repeated doublings