

Year 8 Maths · Unit 8.2 Ratio and Proportion

The big idea. Relationships and equivalence between quantities: similarity and scale factor, ratio, direct proportion, conversion graphs, scale diagrams and map scales.

Scale copies and similarity

- Two shapes are **similar** if one is an enlargement of the other: matching angles are equal and matching lengths are in the same ratio.
- Scale factor** = new length $\div$ old length. A 3 cm side becoming 12 cm has scale factor $12 \div 3 = 4$.
- Use the scale factor to find a missing length: multiply by it going bigger, divide going smaller.
- Stretching in one direction only does **not** give a copy: the shape is distorted.

Ratio

- A **ratio** compares two quantities: 3 : 2. **Proportion** compares a part with the whole: $\frac{3}{5}$.
- Total parts = add the ratio numbers. To share 40 in the ratio 3 : 2: 5 parts, 1 part = 8, so 24 and 16.
- Simplify** a ratio by dividing both by the HCF: 18 : 24 = 3 : 4. Use the same units first: 2 m : 50 cm = 200 : 50 = 4 : 1.
- Equivalent** ratios come from multiplying or dividing both parts by the same number.
- Form **1 : n** (divide by the first): 4 : 10 = 1 : 2.5. Form **n : 1** divides by the second. The unit ratio makes comparing easy.

Ratio and fractions

- Turn a ratio into fractions of the whole: 3 : 2 means $\frac{3}{5}$ and $\frac{2}{5}$.
- Turn a fraction into a ratio: $\frac{1}{4}$ are girls means girls : others = 1 : 3.
- Part of a quantity: $\frac{3}{5}$ of 40 = 24.
- A ratio compares part with part (3 : 2); a fraction compares part with whole ($\frac{3}{5}$).

Direct proportion

- Two quantities are in **direct proportion** if when one doubles so does the other. Their graph is a straight line through the origin.
- Use the **unit value**: 5 pens cost 15 francs, so 1 pen costs 3 francs, and 8 pens cost 24 francs.
- The **constant of proportionality** k is $y \div x$. The rule is **$y = kx$** . If $y = 15$ when $x = 5$, $k = 3$ and $y = 3x$.
- Test a table: if $y \div x$ is the same every time, it is proportional.
- A relationship that does not pass through the origin (for example a taxi with a fixed charge) is **not** proportional.

Conversion graphs

- A conversion graph is a straight line through the origin: for example francs to euros.
- Read across from one axis to the line, then down (or across) to the other. You can convert both ways.
- Draw one from two points (the origin and one known pair) and join them.
- If the line does not go through the origin, the relationship is not proportional.

Scale diagrams and map scales

- A scale **1 : 50** means 1 cm on the drawing is 50 cm in real life.
- Real length = drawing length $\times$ scale. Drawing length = real length $\div$ scale.

- Map scale 1 : 25 000: 4 cm on the map is $4 \times 25\,000 = 100\,000\text{ cm} = 1\text{ km}$.
- Change units sensibly: $100\text{ cm} = 1\text{ m}$ and $100\,000\text{ cm} = 1\text{ km}$.
- Choose a scale that fits the paper and keeps the drawing clear.

Common mistakes

- ✗ “Share 40 in the ratio 3 : 2 as $40 \div 3$ and $40 \div 2$.” ✓ Add the parts (5), find one part (8), then multiply: 24 and 16.
- ✗ “Ratio 2 m : 50 cm is 2 : 50.” ✓ Change to the same units first: $200 : 50 = 4 : 1$.
- ✗ “Ratio and fraction are the same thing.” ✓ 3 : 2 compares part with part; $\frac{3}{5}$ compares part with whole.
- ✗ “Any straight line graph is proportional.” ✓ It must also pass through the origin.
- ✗ “1 : 25 000 means 1 cm is 25 000 m.” ✓ 1 cm is $25\,000\text{ cm} = 250\text{ m}$.