

Year 7 Maths · Unit 7.1 Positive Integers

The big idea. Connections and relationships between positive whole numbers: divisibility, multiples, factors, powers, primes, prime factors, HCF and LCM.

Divisibility tests

- Divisible by **2, 5, 10**: look at the last digit (even; 0 or 5; 0).
- **3** and **9**: add the digits. If the digit sum is a multiple of 3 (or 9), so is the number.
- **4**: last two digits divisible by 4. **8**: last three digits divisible by 8.
- **6**: the number passes both the 2 test and the 3 test.
- Example: 156 is even and $1 + 5 + 6 = 12$, so it is divisible by 6.

Multiples and factors

- A **multiple** is a number times a whole number. The list never ends: multiples of 7 are 7, 14, 21, 28, 35 ...
- **Common multiples** of 4 and 6: 12, 24, 36 ... The smallest is the lowest common multiple.
- **Factors** divide exactly. Find them in pairs from the outside in: $36 = 1 \times 36, 2 \times 18, 3 \times 12, 4 \times 9, 6 \times 6$.
- Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36.
- Only **square numbers** have an odd number of factors (one pair repeats, like 6×6).

Powers and roots

- Square numbers to 100: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100. First five cubes: 1, 8, 27, 64, 125.
- $3^2 = 3 \times 3 = 9$ and $2^3 = 2 \times 2 \times 2 = 8$. (2^3 is not 6.)
- A **square root** undoes squaring: $\sqrt{49} = 7$. A **cube root** undoes cubing: $\sqrt[3]{27} = 3$.
- Mixed: $\sqrt{64} + 3^2 = 8 + 9 = 17$.

Sequences

- A **term-to-term rule** takes you from one term to the next: add 5 (4, 9, 14, 19), double (3, 6, 12, 24), halve (64, 32, 16, 8).
- Describe a rule in words: say where it starts and what happens each time.
- **Triangle numbers**: 1, 3, 6, 10, 15, 21 ... The gap grows by 1 each time (+2, +3, +4 ...).
- Square numbers: 1, 4, 9, 16 ... gaps 3, 5, 7 grow by 2.

Prime numbers

- A **prime** has exactly two factors: 1 and itself. A **composite** number has more than two. **1 is neither**.
- 2 is the only even prime.
- **Sieve of Eratosthenes**: cross out multiples of 2, 3, 5, 7 ... what is left (apart from 1) is prime. Primes to 30: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.
- Test a number by trying primes up to its square root. $91 = 7 \times 13$, so 91 is not prime.

Prime factors

- Every whole number above 1 is a **product of primes** in exactly one way.
- Factor tree: split into a factor pair, keep splitting until every branch is prime.
- Expanded form $36 = 2 \times 2 \times 3 \times 3$. Index form $36 = 2^2 \times 3^2$.
- Ladder method: divide by the smallest prime that works until you reach 1. $72 = 2^3 \times 3^2$.
- A number divides another if its primes are all inside the other's: $12 = 2^2 \times 3$ divides $60 = 2^2 \times 3 \times 5$.

HCF and LCM

- **HCF** (highest common factor) = SHARING: biggest group size with none left over. $\text{HCF}(12, 18) = 6$.
- **LCM** (lowest common multiple) = LINING UP: the first time events meet again. $\text{LCM}(4, 6) = 12$.
- Venn method with prime factors: **overlap** gives the HCF, **everything** gives the LCM. $36 = 2^2 \times 3^2$ and $60 = 2^2 \times 3 \times 5$: $\text{HCF} = 2 \times 2 \times 3 = 12$, $\text{LCM} = 2 \times 2 \times 3 \times 3 \times 5 = 180$.
- Answer in the words of the problem: buses every 12 and 18 minutes meet again after 36 minutes.

Common mistakes

- ✗ “1 is a prime number.” ✓ 1 has only one factor, so it is neither prime nor composite.
- ✗ “ $2^3 = 6$.” ✓ $2^3 = 2 \times 2 \times 2 = 8$. The power says how many 2s are multiplied.
- ✗ “Multiples and factors are the same.” ✓ Factors of 12 are smaller or equal (1, 2, 3, 4, 6, 12). Multiples are bigger or equal (12, 24, 36 ...).
- ✗ “Use HCF for a bus timetable.” ✓ Events that repeat and meet again need the LCM. Sharing into equal groups needs the HCF.
- ✗ “A prime factor tree can end on 9.” ✓ Keep splitting until every branch is prime: $9 = 3 \times 3$.