UNIT 7.1

Positive Integers

Revision slides: everything you need for the test, one idea per slide.

Lorna Brown

UNIT 7.1 · REVISE

Divisibility tests

Divisible by **2**, **5**, **10**: look at the last digit (even; 0 or 5; 0).

3 and **9**: add the digits. If the digit sum is a multiple of 3 (or 9), so is the number.

4: last two digits divisible by 4. **8**: last three digits divisible by 8.

6: the number passes both the 2 test and the 3 test.

Example: 156 is even and 1 + 5 + 6 = 12, so it is divisible by 6.

Multiples and factors (1 of 2)

A **multiple** is a number times a whole number. The list never ends: multiples of 7 are 7, 14, 21, 28, 35 ...

Common multiples of 4 and 6: 12, 24, 36 ... The smallest is the lowest common multiple.

Multiples and factors (2 of 2)

Factors divide exactly. Find them in pairs from the outside in: 36 = 1 × 36, 2 × 18, 3 × 12, 4 × 9, 6 × 6.

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36.

Only **square numbers** have an odd number of factors (one pair repeats, like 6 × 6).

UNIT 7.1 · REVISE

Powers and roots

Square numbers to 100: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100. First five cubes: 1, 8, 27, 64, 125.

$3^2 = 3 \times 3 = 9$ and $2^3 = 2 \times 2 \times 2 = 8$. ($2^3$ is not 6.)

A **square root** undoes squaring: $\sqrt{49} = 7$. A **cube root** undoes cubing: $\sqrt[3]{27} = 3$.

Mixed: $\sqrt{64} + 3^2 = 8 + 9 = 17$.

UNIT 7.1 · REVISE

Sequences

A **term-to-term rule** takes you from one term to the next: add 5 (4, 9, 14, 19), double (3, 6, 12, 24), halve (64, 32, 16, 8).

Describe a rule in words: say where it starts and what happens each time.

Triangle numbers: 1, 3, 6, 10, 15, 21 ... The gap grows by 1 each time (+2, +3, +4 ...).

Square numbers: 1, 4, 9, 16 ... gaps 3, 5, 7 grow by 2.

UNIT 7.1 · REVISE

Prime numbers

A **prime** has exactly two factors: 1 and itself. A **composite** number has more than two. **1 is neither**.

2 is the only even prime.

Sieve of Eratosthenes: cross out multiples of 2, 3, 5, 7 … what is left (apart from 1) is prime. Primes to 30: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

Test a number by trying primes up to its square root. 91 = 7 × 13, so 91 is not prime.

UNIT 7.1 · REVISE

Prime factors

Every whole number above 1 is a **product of primes** in exactly one way.

Factor tree: split into a factor pair, keep splitting until every branch is prime.

Expanded form $36 = 2 \times 2 \times 3 \times 3$. Index form $36 = 2^2 \times 3^2$.

Ladder method: divide by the smallest prime that works until you reach 1. $72 = 2^3 \times 3^2$.

A number divides another if its primes are all inside the other's: $12 = 2^2 \times 3$ divides $60 = 2^2 \times 3 \times 5$.

UNIT 7.1 · REVISE

HCF and LCM

HCF (highest common factor) = SHARING: biggest group size with none left over. HCF(12, 18) = 6.

LCM (lowest common multiple) = LINING UP: the first time events meet again. LCM(4, 6) = 12.

Venn method with prime factors: **overlap** gives the HCF, **everything** gives the LCM. $36 = 2^2 \times 3^2$ and $60 = 2^2 \times 3 \times 5$: HCF = 2 × 2 × 3 = 12, LCM = 2 × 2 × 3 × 3 × 5 = 180.

Answer in the words of the problem: buses every 12 and 18 minutes meet again after 36 minutes.

WATCH OUT · MISCONCEPTIONS

Don’t fall for these

✕ “1 is a prime number.”	✓ 1 has only one factor, so it is neither prime nor composite.
✕ “$2^3 = 6$.”	✓ $2^3 = 2 \times 2 \times 2 = 8$. The power says how many 2s are multiplied.
✕ “Multiples and factors are the same.”	✓ Factors of 12 are smaller or equal (1, 2, 3, 4, 6, 12). Multiples are bigger or equal (12, 24, 36 ...).
✕ “Use HCF for a bus timetable.”	✓ Events that repeat and meet again need the LCM. Sharing into equal groups needs the HCF.
✕ “A prime factor tree can end on 9.”	✓ Keep splitting until every branch is prime: $9 = 3 \times 3$.

TEST YOURSELF

Test yourself

1. Which number is divisible by 9?

Answer 4 563. 4 + 5 + 6 + 3 = 18, which is in the 9 times table.

2. Is 3 124 divisible by 4?

Answer Yes, the last two digits 24 are divisible by 4. 24 ÷ 4 = 6, so 3 124 is divisible by 4.

3. Which number is prime?

Answer 53. 51 = 3 × 17, 57 = 3 × 19 and 91 = 7 × 13. 53 has only the factors 1 and 53.

4. Why is 1 not a prime number?

Answer It has only one factor. A prime needs exactly two factors.